Perfect complements combined with perfect substitutes

An individual has the following utility function:

$$u(x_1, x_2) = x_1 + \min(x_1, x_2)$$

Derive the Marshallian demands for both goods.

Solutions

The utility function combines a perfect substitutes function with a perfect complements function. Therefore, on one hand, if the price of x_1 is sufficiently low, the individual will only demand x_1. On the other hand, if the price of x_2 is sufficiently low, the individual will have to demand $x_1 = x_2$ since x_2 is inside the minimum function. The utility of demanding only x_1 is:

$$U = \frac{M}{p_1} + \min(M/p_1, 0) = \frac{M}{p_1}$$

On the other hand, if the individual demands $x_2 = x_1$:

$$M = x_2(p_1 + p_2)$$

$$x_1 = x_2 = \frac{M}{p_1 + p_2}$$

$$U = \frac{M}{p_1 + p_2} + \min(\frac{M}{p_1 + p_2}, \frac{M}{p_1 + p_2}) = 2\frac{M}{p_1 + p_2}$$

The comparison will then be between demanding only x_1 or demanding both goods in the same amount. In terms of utility, we compare the following:

$$2\frac{M}{p_1 + p_2} \ ? \ \frac{M}{p_1}$$

We divide both terms by M:

$$\frac{2}{p_1 + p_2} \ ? \ \frac{1}{p_1}$$

We multiply by p_1 and by $p_1 + p_2$:

$$2p_1 \ ? \ p_1 + p_2$$

$$p_1 \ ? \ p_2$$

This leads us to a comparison between the prices of the goods. If $p_1 > p_2$, then both goods will be demanded. Conversely, if the opposite is true, only x_1 will be demanded.

$$x_1 = \begin{cases} \frac{M}{p_1} & p_1 < p_2 \\ \frac{M}{p_1 + p_2} & p_1 > p_2 \end{cases}$$

$$x_2 = \begin{cases} 0 & p_1 < p_2 \\ \frac{M}{p_1 + p_2} & p_1 > p_2 \end{cases}$$

Finally, in the case where $p_1 = p_2$, the individual can choose to demand only x_1 or any combination of x_1 and x_2, since they will be indifferent.